Killara
High School

Student Number

2024 **Year 12 Trial Examination**

Mathematics Advanced

12/08/2024

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using blue or black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- No white-out may be used

Total Marks: 100

Section I - 10 marks (pages 3 - 7)

- Allow about 15 minutes for this section

Section II - 90 marks (pages 8 - 39)

- Allow about 2 hours and 45 minutes for this section

This question paper must not be removed from the examination room.

This assessment task constitutes 30% of the course.

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section.

Use the multiple-choice sheet for Questions 1–10.

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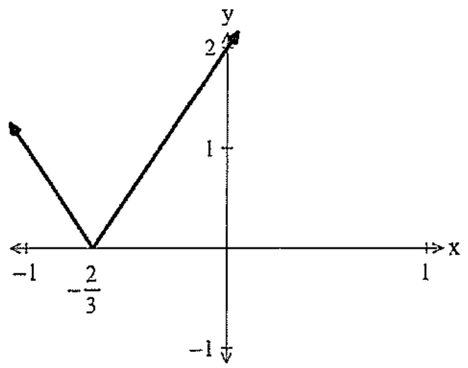
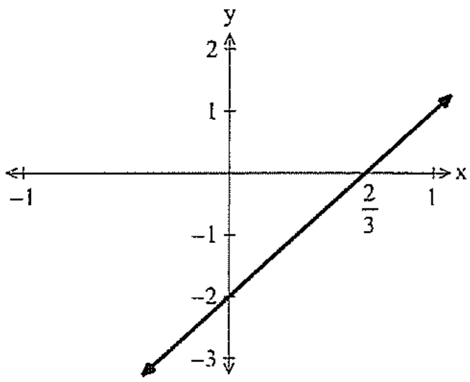
1 Which diagram is the correct sketch of $y = |3x - 2|$?

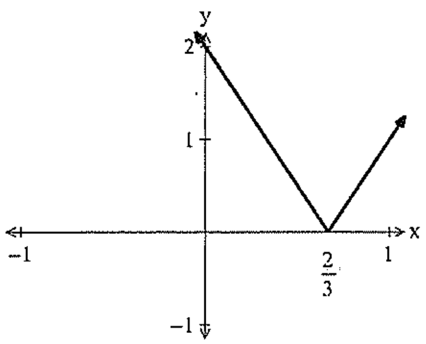
A.



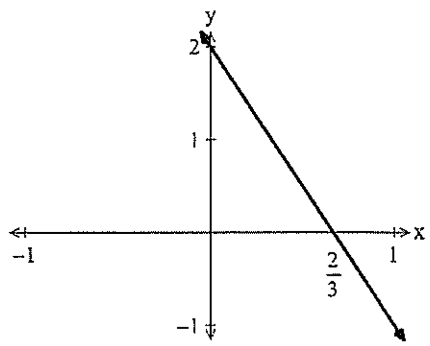
B.



C.



D.



2 If $f(x) = 3x^2$ and $g(x) = \frac{3}{x}$, which of the following statements is correct?

A. f and g are both odd functions

B. f is an even function and g is an odd function

C. f and g are both even functions

D. f and g are neither even nor odd functions

3 A bag contains 4 blue coloured marbles and 6 red coloured marbles. Three marbles are selected at random without replacement.
What is the probability that at least one of the marbles selected is blue?

A. $\frac{1}{6}$

B. $\frac{1}{2}$

C. $\frac{5}{6}$

D. $\frac{29}{30}$

4 What is the domain of $f(x) = \log_2(1 - 2x)$?

A. $x > \frac{1}{2}$

B. $x > -\frac{1}{2}$

C. $x < \frac{1}{2}$

D. $x < -\frac{1}{2}$

5 The number of students in the seven schools in a specific school zone are given below:

$$218, 265, 284, 301, 336, 348, 383$$

Suppose that the number 383 **from this list** changes to 432. What will happen to the mean and median?

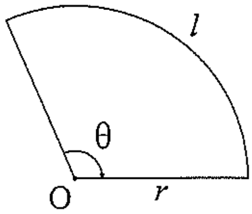
A. The mean increases and the median increases.

B. The mean increases and the median stays the same.

C. The mean stays the same and the median increases.

D. The mean and the median stay the same.

6 It is given that in a sector, the ratio of the length of arc (l) to radius (r) is $\frac{\pi}{a}$.



Which of the following is the correct expression for finding the size of the central angle (θ) in degrees?

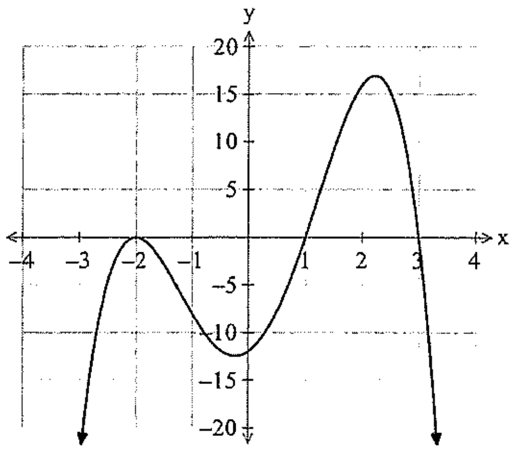
A. $180a$

B. $\dfrac{180}{a}$

C. $\dfrac{180\pi}{a}$

D. πa

7 Which of the following function could be the given polynomial graph?



A. $y = (x+2)^2(x-1)(x-3)$

B. $y = -(x+2)^2(x-1)(x+3)$

C. $y = (x+2)(x-1)^2(3-x)$

D. $y = (x+2)^2(x-1)(3-x)$

8 Let $h(x) = \frac{f(x)}{g(x)}$ where

$$f(2) = 4 \qquad g(2) = -2$$

$$f'(2) = \frac{1}{2} \qquad g'(2) = 2$$

What is the gradient of the tangent to the graph of $y = h(x)$ at $x = 2$?

A. $-\frac{9}{4}$

B. $\frac{9}{4}$

C. $\frac{9}{16}$

D. -2

9 A function $f(x)$ is such that

$$\int_{-k}^{k} f(x)dx = 2\int_{0}^{k} f(x)dx$$

Which of the following could $f(x)$ be?

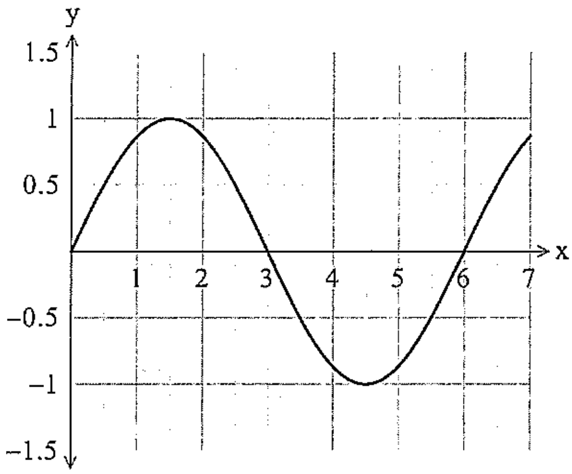
A. $f(x) = \sin 2x$

B. $f(x) = \cos 2x$

C. $f(x) = \tan 2x$

D. $f(x) = \text{cosec}2x$

10 The graph of $y = \sin\left(\frac{\pi x}{3}\right)$ where $x \in [0,7]$ is shown below:



Which of the following is the solution to the inequality?

$$\sin\left(\frac{\pi x}{3}\right) + \frac{1}{2} \geq 0$$

A. $x \in \left[0, \frac{1}{2}\right] \cup \left[\frac{5}{2}, 7\right]$

B. $x \in \left[\frac{1}{2}, \frac{5}{2}\right]$

C. $x \in \left[0, \frac{7}{2}\right] \cup \left[\frac{11}{2}, 7\right]$

D. $x \in \left[\frac{7}{2}, \frac{11}{2}\right]$

End of Section I

Do NOT write in this area.

Question 11 (2 marks)

Solve for x. **2**

$$6(2x - 3) = 8 - 2(3x + 1)$$

Question 12 (3 marks)

The population of a town in NSW has shown a linear decline in the years 2011 to 2019. In 2011 the population was 34300 people. In 2019 it was 27740 people.

(a) Write a linear equation expressing the population of the town, P, as a function of t, the number of years since 2011. **2**

(b) If the town is still experiencing a linear decline, what will the population be in 2025? **1**

Question 13 (2 marks)

Evaluate 2

$$\int_1^3 (2x+1)^2\,dx$$

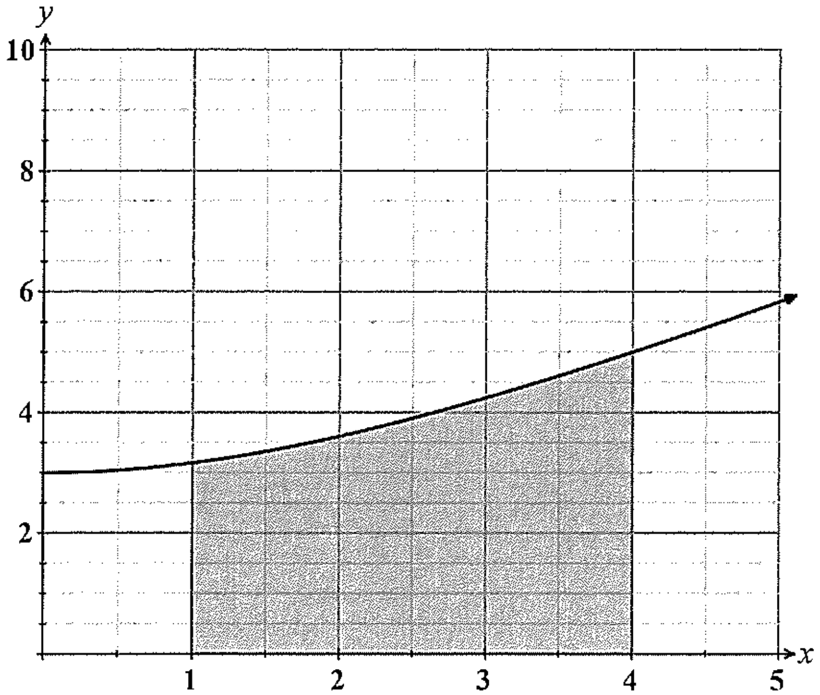
Question 14 (3 marks)

Given that $E\ (X) = 2.5$, find a and b. 3

x	1	2	3	4
$P(X = x)$	0.3	$\boldsymbol{a}$	$\boldsymbol{b}$	0.2

Question 15 (2 marks)

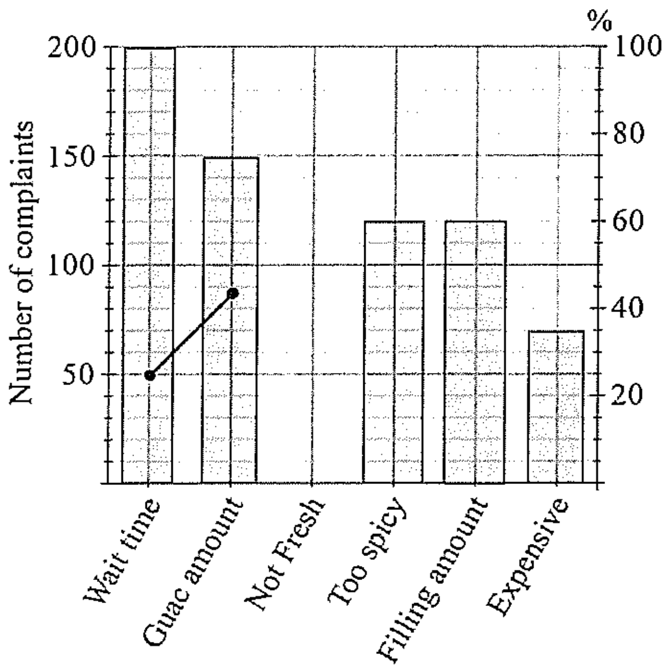
Given the function $f(x) = \sqrt{x^2 + 9}$, use the trapezoidal rule and four function values to find the area under the curve between $x = 1$ and $x = 4$. Correct your answer to 2 decimal places. **2**



Question 16 (3 marks)

At Kyan's Burritos, the owners regularly ask their customers if and why they are not happy with their burritos. Kyan began to draw a pareto chart to display this information but left it incomplete. The partially completed Pareto Chart is shown below:

Reasons for customer dissatisfaction



Type of complaint	Frequency	Cumulative frequency	Cumulative percentage (%)
Wait time	200	200	25%
Guac amount	150	350	43.75%
Not fresh	*A*		61.25%
Too spicy	120	610	76.25%
Filling amount	120	730	91.25%
Expensive	70	800	100%

Show that the value of *A* is 140. **3**

Hence, complete the table and the pareto chart above.

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DO NOT write in this area.

Question 17 (2 marks)

Find the equation of the tangent to the curve $y = \sin x$ at the point $\left(\frac{\pi}{3}, \frac{1}{2}\right)$. Leave in the form of the $ay + bx + c = 0$, where a, b and c are exact values. **2**

Question 18 (2 marks)

The first term of an arithmetic series is 7, the common difference is 2 and the sum of the first n terms is 247. Find the value of n. **2**

Question 19 (6 marks)

A population of bacteria is modelled by the function $P\ (t) = P_0 \times e^{kt}$, where $P(t)$ is the population at time t (in hours), P_0 is the initial population, and k is the constant growth rate.

(a) Given that the initial population P_0 is 100 bacteria and the population triples every 4 hours, show that value of $k = \frac{ln3}{4}$. **2**

(b) Determine the population of bacteria after 12 hours. **2**

(c) Find the time it takes for the population to reach 100,000 bacteria. Leave your answer to the nearest minute. **2**

Question 20 (3 marks)

Given the function $f(x) = 2x^3 - \frac{5}{2}x^2 - 4x + 2$. Determine the interval(s) where $f(x)$ is decreasing. **3**

Question 21 (3 marks)

Given the following two functions $f(x) = x^2 - 3$ and $g(x) = \sqrt{2 - x}$. **3**

Find the domain and range of $f(g(x))$.

Question 22 (5 marks)

(a) Prove **2**

$$\frac{\sec^2(2\theta)}{\sec^2(2\theta) - 1} = \operatorname{cosec}^2(2\theta).$$

(b) Hence, solve $\dfrac{\sec^2(2\theta)}{\sec^2(2\theta) - 1} = 2$ where $0 \le \theta \le \pi$. **3**

DO NOT write in this area.

Question 23 (4 marks)

A ship sets sail from **point P** on a bearing of 060° for 150 km to **point Q**. It then changes course and sails on a bearing of 160° for 200 km to **point R**.

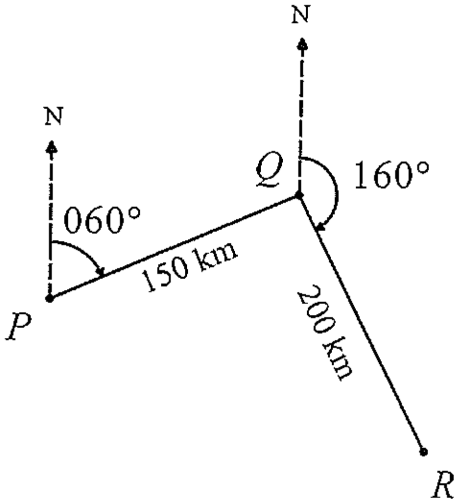

Diagram not to scale

(a) Determine the distance from point **P** to point **R**. Correct your answer to 2 decimal places. **2**

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(b) Determine the bearing the ship must take to return directly to point **P** from point **R**. Correct your answer to the nearest degree. **2**

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Question 24 (5 marks)

A financial analyst is studying the relationship between the number of years of experience, n (in years) and the salary, s (in thousands of dollars) of employees in a certain industry. The analyst collects data from a sample of employees and records their years of experience and corresponding salaries as follows:

Years of Experience (n)	Salary (s) (in \$1000s)
1	65
3	70
5	80
7	100
9	130

(a) Calculate the least squares regression line for this data. **2**

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(b) Interpret the slope and intercept of the regression line in the context of this problem. **2**

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(c) Why is this line **NOT** useful for predicting the salary for a person who has 12 years of work experience? **1**

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Question 25 (3 marks)

The scores of a Year 11 economics examination are shown in the back-to-back stem and leaf plot below for classes 11A and 11B. **3**

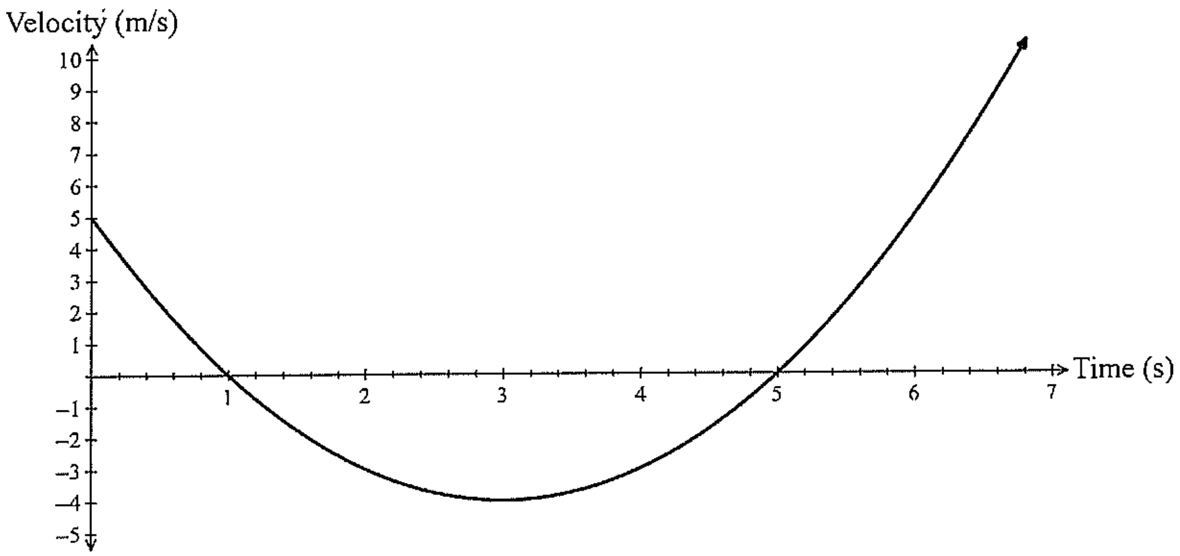
11 A		11 B
8 7 5	5	9
9 8 8 8 4 2	6	4 4
7 5 2	7	0 3 5 7
3 1	8	2 5 6 7 8
4	9	1 2

Mrs Cartwright claims that class 11B did better in the examination than class 11A.

Do you agree with Mrs Cartwright? Justify your answer by referring to the median and skewness of the two sets of scores.

Question 26 (5 marks)

The velocity-time graph shows how a particle travels during a period of 7 seconds. Initially, the particle is at the origin and travelling at 5 m/s to the right. The graph has two horizontal intercepts at $t = 1$ and $t = 5$. It also has a turning point at $t = 3$.



(a) Explain the meaning of the horizontal intercept at $t = 1$. **1**

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(b) When the acceleration is not zero, what is the direction of the acceleration? **2**

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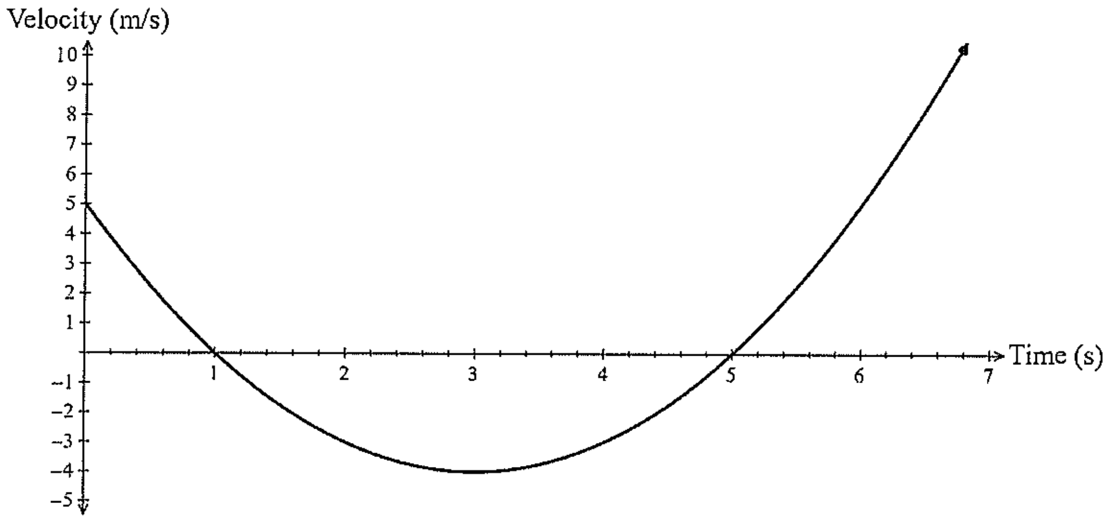
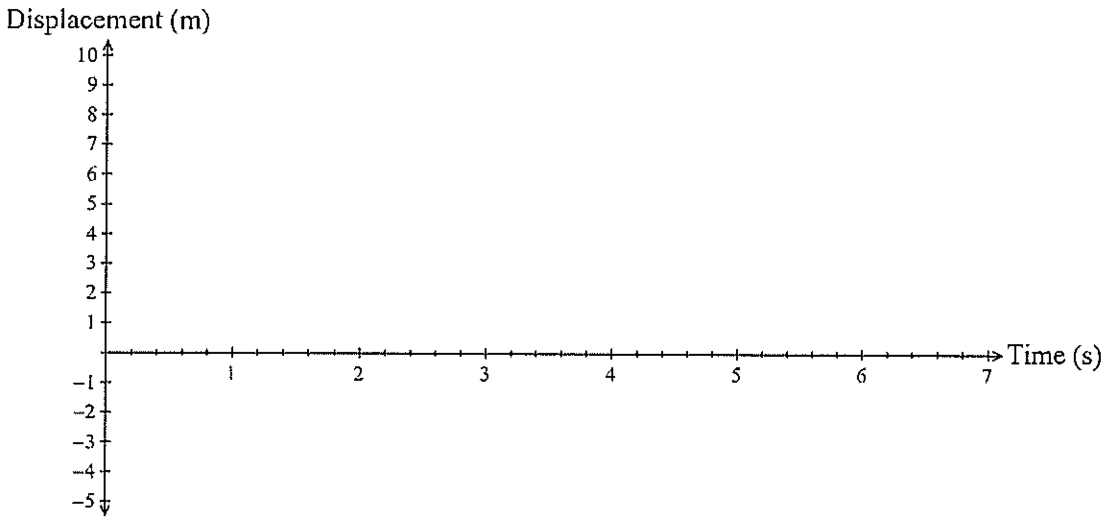
Question 26 continues page 23

Question 26 (continued)

(c) Hence sketch the displacement-time graph below. The displacement is 0 when $t = 2.2\ sec$ and $t = 6.8\ sec$. **2**





Question 27 (3 marks)

Solve the following equation: **3**

$$x - 2x^2 + x^3 - 2x^4 + x^5 - 2x^6 + \cdots = -\frac{2}{5}$$

Where $-1 \leq x \leq 1$

End of Booklet 1 Section II

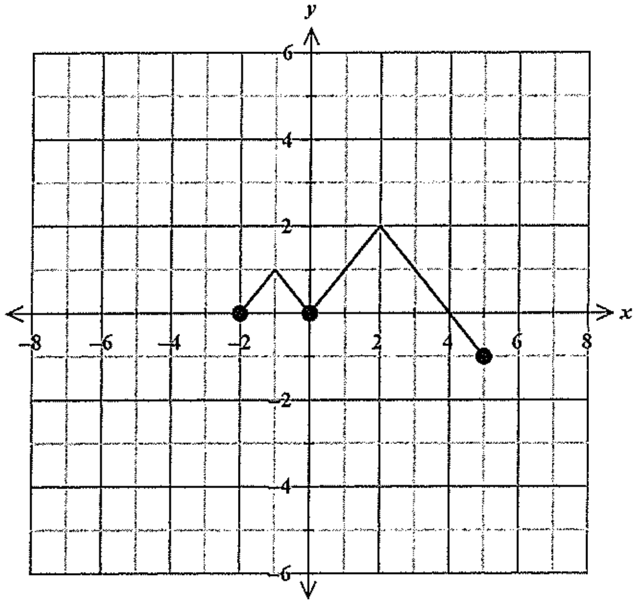
Question 28 (2 marks)

Given that $\frac{dy}{dx} = \cos\left(x - \frac{\pi}{4}\right)$ and $y = 2$ when $x = \frac{3\pi}{4}$, find y in terms of x. **2**

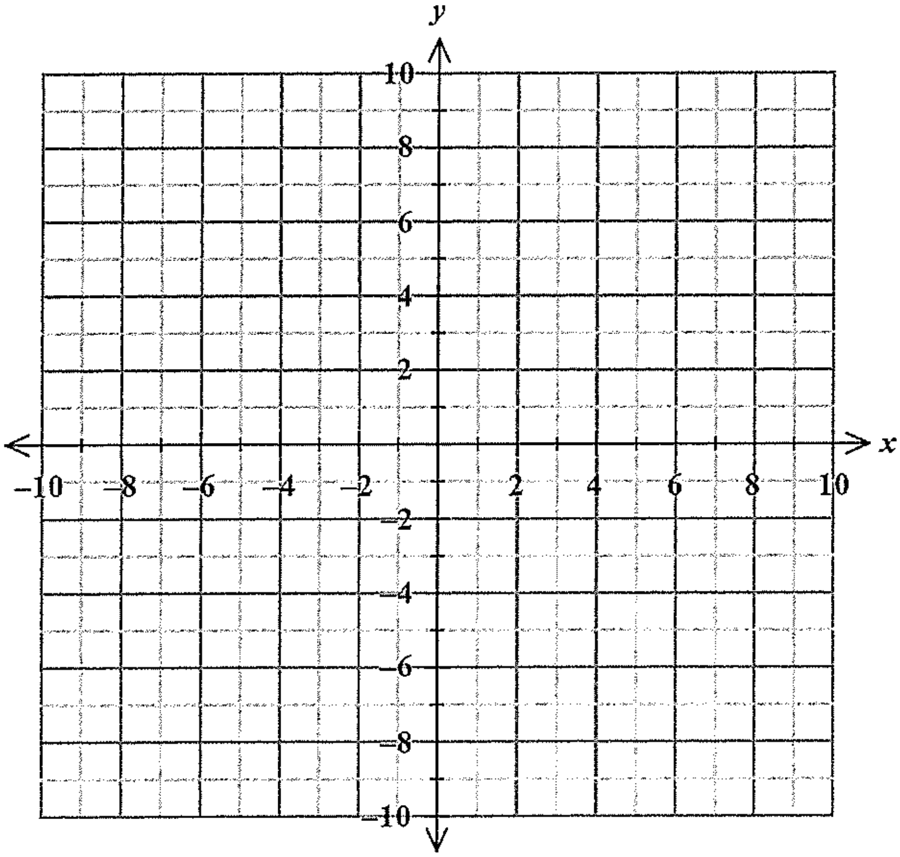
Do NOT write in this area.

Question 29 (2 marks)

Given the following function $f(x)$.



Sketch $2f\left(-\frac{1}{2}x\right)$ on the axis below. **2**



Question 30 (6 marks)

The quantity Q in mL of a certain chemical in the body varies during the day and is given by the formula $Q(t) = 4 + 3\cos\left(\frac{\pi t}{4}\right)$

where t measures hours from midnight.

(a) Find the period in hours of the function Q. **1**

(b) At what time or times of the day is the quantity a minimum? **2**

(c) What is the minimum quantity of the chemical the body will contain? **1**

(d) A hospital patient requires a pill to temporarily boost the amount of the chemical whenever the quantity in his body falls to 1.5 units. At what time will a nurse have to wake the patient to give him his first pill of the day? Give your answer to nearest minute. **2**

Do NOT write in this area.

Question 31 (4 marks)

Given that $y = x^2 \ln(x)$

(a) Show that **2**

$$\frac{dy}{dx} = 2x\ln(x) + x$$

(b) Hence, show that **2**

$$\int x\ln(x)\,dx = \frac{x^2\ln(x)}{2} - \frac{x^2}{4} + C$$

Question 32 (4 marks)

The first two terms of an infinite geometric sequence are $T_1 = 20$ and $T_2 = 16\sin^2\theta$, where $0 < \theta < 2\pi$, $\theta \neq \pi$.

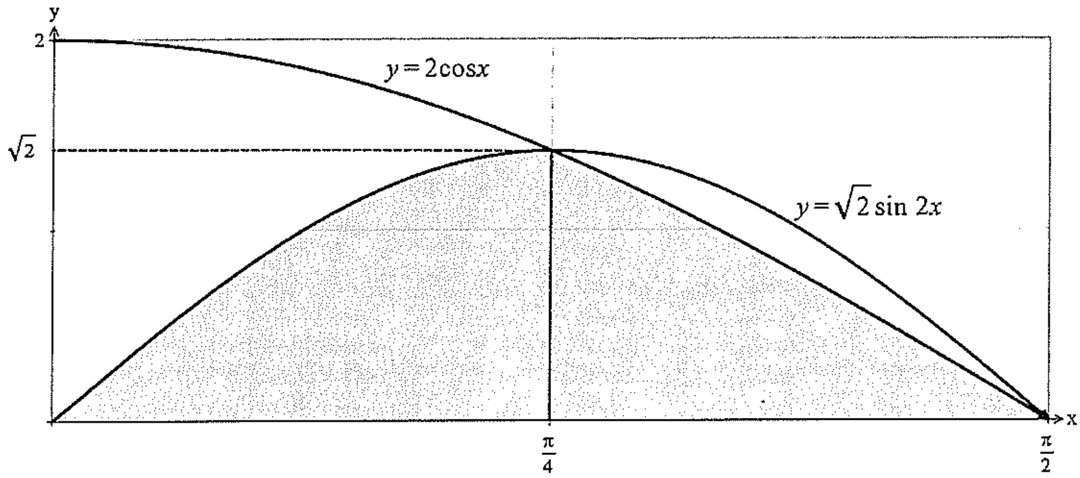
(a) Find the range of the ratio (r) in this geometric sequence. **2**

(b) By developing an expression for the sum of the infinite sequence, find the values of θ which give the greatest sum. **2**

Question 33 (3 marks)

The diagram below shows the graphs of the functions $y = 2\cos x$ and $y = \sqrt{2}\sin 2x$ between $x = 0$ and $x = \frac{\pi}{2}$. **3**

The two graphs intersect at $x = \frac{\pi}{4}$ and $x = \frac{\pi}{2}$.



Find the area of the shaded region. Leave your answer in exact form.

Question 34 (3 marks)

A flood insurance company determines that N, the number of claims received in a month, is a random variable with 3

$$P(N = n) = \frac{2}{3^{n+1}}, \text{ for } n = 0, 1, 2, \dots$$

The numbers of claims received in different months are independent.

In any consecutive two-month period, calculate the probability that more than one claim will be received, given that zero claims were received at least one of the two months.

Do NOT write in this area.

Question 35 (5 marks)

Melinda visits a bank and makes a single deposit of $\$Q$. The annual interest rate is 3.5%.

(a) Melinda wishes to withdraw \$8000 at the end of each year for a period of n years. Show that an expression for the minimum value of Q is **3**

$$\frac{8000}{1.035}+\frac{8000}{1.035^2}+\frac{8000}{1.035^3}+\cdots+\frac{8000}{1.035^n}$$

Question 35 continues on page 37

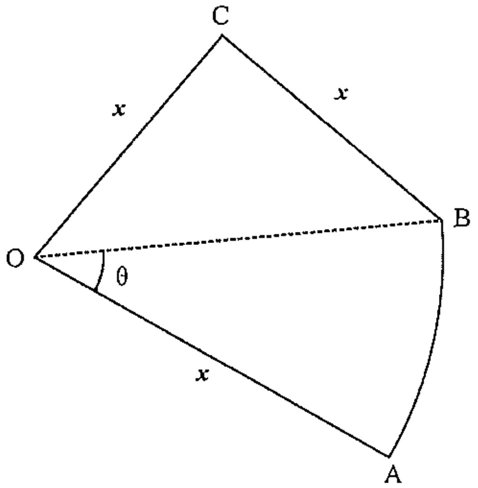
Question 35 (continued)

(b) Hence, or otherwise, find the minimum value Q that would permit Melinda to withdraw annual amounts of \$8000 indefinitely. Give your answer to the nearest dollar. **2**

Do NOT write in this area.

Question 36 (5 marks)

The diagram below shows a large bio-diversity precinct the council is planning to build inside a large park. OAB is a sector with centre O, and radius x kilometres. Arc AB subtends an angle of θ radians at O. The equilateral triangle BCO adjoins the sector. **5**



The perimeter of the precinct as shown in the diagram is given to be $(12 - 2\sqrt{3})$ kilometres.

Calculate the maximum area of the precinct $(OABC)$. Leave your answer in exact form.

Extra writing space is provided on page 39

Do NOT write in this area.

Question 36 (extra writing space)

End of paper

Killara High School

Solutions + Marking Scheme

Student Number

marker's feedback

2024 Year 12 Trial Examination

Mathematics Advanced

Section II Answer Booklet 1

12/08/2024

Section II

90 Marks
Attempt Questions 11–36
Allow about 2 hours and 45 minutes for this section

Booklet 1 – Attempt Questions 11–27 (56 marks)
Booklet 2 – Attempt Questions 28–36 (34 marks)

Q	Marks
11	/2
12	/3
13	/2
14	/2
15	/3
16	/2
17	/2
18	/6
19	/3
20	/3
21	/5
22	/4
23	/3
24	/5
25	/3
26	/5
27	/3
Total	**/56**

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on page 25 & 26.

  If you use this space, clearly indicate which question you are answering.

Please turn over

Killara High School

Student Number

Mathematics Advanced

Section I – Multiple Choice Answer Sheet
Use this multiple-choice answer sheet for questions 1 – 10.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.

correct

A ⊗ B ⊗ C ○ D ○

Start Here →

1. A ○ B ○ C ● D ○
2. A ○ B ● C ○ D ○
3. A ○ B ○ C ● D ○
4. A ○ B ○ C ● D ○
5. A ○ B ● C ○ D ○
6. A ○ B ● C ○ D ○
7. A ○ B ○ C ○ D ●
8. A ● B ○ C ○ D ○
9. A ○ B ● C ○ D ○
10. A ○ B ○ C ● D ○

Question 11 (2 marks)

Solve for x. **2**

$$6(2x - 3) = 8 - 2(3x + 1)$$

$12x - 18 = 8 - 6x - 2$

$18x = 24$

$\therefore x = \frac{4}{3}$

1 mark: Correct working
1 mark: Correct answer with simplified fraction form.

Most students answered correctly.
Some students incorrectly did $\frac{18}{24}$ or with out simplifying the fraction.

Question 12 (3 marks)

The population of a town in NSW has shown a linear decline in the years 2011 to 2019. In 2011 the population was 34300 people. In 2019 it was 27740 people.

(a) Write a linear equation expressing the population of the town, P, as a function of t, the number of years since 2011. **2**

Poorly done. Lots of students modelled using exponential model.

$P = 34300 + kt$

when $t = 8$, $P = 27740$

$27740 = 34300 + 8k$

$k = -820$

$\therefore P = -820t + 34300$

1 mark: Correct working finding the declining rate.
1 mark: Correct equation

(b) If the town is still experiencing a linear decline, what will the population be in 2025? **1**

when $t = 14$

$P = -820 \times 14 + 34300$

$= 22\,820$

1 mark: Correct answer with working.

Some students use $t = 2011$ and $t = 2025$. and made carry-on error from part a), ECF was awarded only when there is no other mistake.

Question 13 (2 marks)

Evaluate **2**

$$\int_1^3 (2x+1)^2\,dx$$

$= \frac{1}{2\times 3}\left[(2x+1)^3\right]_1^3$

$= \frac{1}{6}\left[(2\times3+1)^3 - (2+1)^3\right]$

$= \frac{158}{3}$

1 mark: Correct integration

1 mark: Correct answer

Generally done well. Some students didn't apply the reverse chain rule correctly by adding a fudge factor or used $\int (ax+b)^n\,dx$ incorrectly.

Question 14 (3 marks)

Given that $E(X) = 2.5$, find a and b. **3**

x	1	2	3	4
$P(X = x)$	0.3	a	b	0.2

$a + b + 0.5 = 1$

$a + b = 0.5$ (1)

$0.3 + 2a + 3b + 4\times 0.2 = 2.5$

$2a + 3b = 1.4$ (2)

$2a + 3(0.5 - a) = 1.4$

$2a + 1.5 - 3a = 1.4$

$-a = -0.1$

$\therefore a = 0.1$

$b = 0.4$

1 mark: Correct setting for two equations

1 mark: Correct working

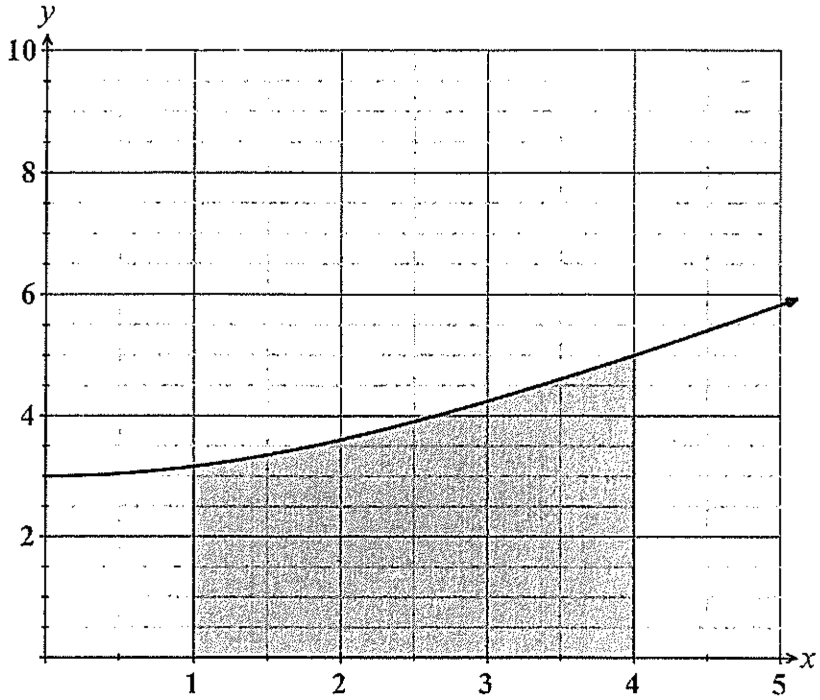
1 mark: Correct answer for a and b.

Generally done well, most students can establish two simultaneous equations using discrete random variable properties

Some let a+b=0.5 without explanation, in those case, 1 mark was taken.

Question 15 (2 marks)

Given the function $f(x) = \sqrt{x^2 + 9}$, use the trapezoidal rule and four function values to find the area under the curve between $x = 1$ and $x = 4$. Correct your answer to 2 decimal places. 2



1 mark: Correct setting

1 mark: Correct answer

$$\text{Area} = \frac{1}{2}\left(f(1) + 2(f(2) + f(3)) + f(4)\right)$$

$$= \frac{1}{2}\left(\sqrt{10} + 2(\sqrt{13} + \sqrt{18}) + 5\right)$$

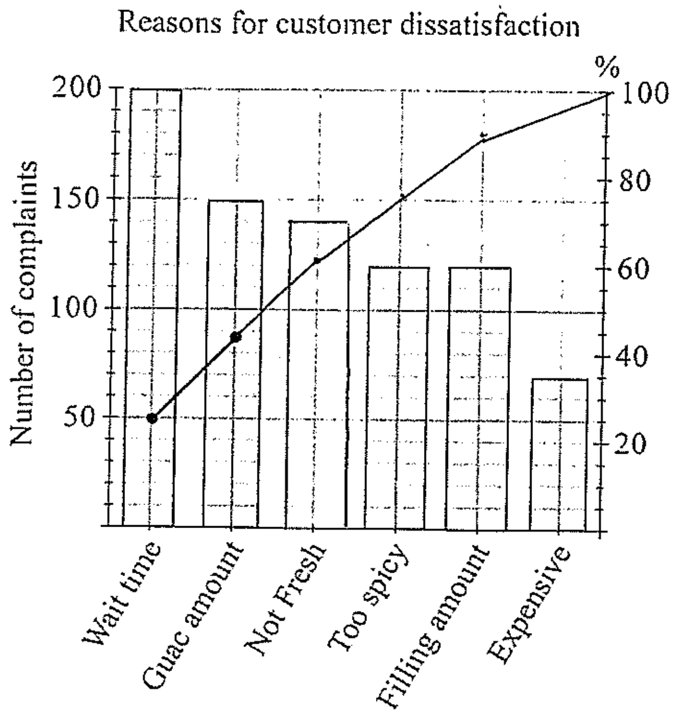
$$\approx 11.93\ u^2$$

Generally done well. Some students misinterpreted as four sub-intervals. If the working was correct afterwards, 1 mark was awarded.

* Most students answered this well. The most common error was not completing the pareto chart.

Question 16 (3 marks)

At Kyan's Burritos, the owners regularly ask their customers if and why they are not happy with their burritos. Kyan began to draw a pareto chart to display this information but left it incomplete. The partially completed Pareto Chart is shown below:



1 mark: Correct working for ~~show value~~ finding A
1 mark: Correct Column graph for Not Fresh
1 mark: Correct Pareto Chart.

Type of complaint	Frequency	Cumulative frequency	Cumulative percentage (%)
Wait time	200	200	25%
Guac amount	150	350	43.75%
Not fresh	*A*	490	61.25%
Too spicy	120	610	76.25%
Filling amount	120	730	91.25%
Expensive	70	800	100%

Show that the value of *A* is 140. **3**

Hence, complete the table and the pareto chart above.

$800 \times 61.25\% = 490$

$\therefore A = 490 - 350 = 140$

Do NOT write in this area.

Question 17 (2 marks)

Find the equation of the tangent to the curve $y = \sin x$ at the point $\left(\frac{\pi}{3}, \frac{1}{2}\right)$. Leave in the ~~form of the $y = mx + c$~~, where m and c are exact values. general form of the $ay + bx + c = 0$

$\frac{dy}{dx} = \cos x$

when $x = \frac{\pi}{3}$

$\frac{dy}{dx} = \frac{1}{2}$

1 mark for finding the gradient of the tangent

$y - y_1 = m(x - x_1)$

$y - \frac{1}{2} = \frac{1}{2}\left(x - \frac{\pi}{3}\right)$

$0 = y - \frac{x}{2} - \frac{1}{2} + \frac{\pi}{6}$

1 mark for getting to atleast this point

* students found the derivative but some could not find the equation of the tangent.

Question 18 (2 marks)

The first term of an arithmetic series is 7, the common difference is 2 and the sum of the first n terms is 247. Find the value of n. 2

$a = 7$, $d = 2$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$2 \times 247 = n(14 + 2(n-1))$

$494 = 14n + 2n^2 - 2n$

$2n^2 + 12n - 494 = 0$

$n^2 + 6n - 247 = 0$

$(n+19)(n-13) = 0$

$n = -19,\ n = 13$

$\therefore n = 13$

1 mark: Correct workin

1 mark: Correct answe

* Generally answered well. Some students used $T_n = a + (n-1)d$ instead of the sum of series.

Question 19 (6 marks)

A population of bacteria is modelled by the function $P(t) = P_0 \times e^{kt}$, where $P(t)$ is the population at time t (in hours), P_0 is the initial population, and k is the constant growth rate.

(a) Given that the initial population P_0 is 100 bacteria and the population triples every 4 hours, show that value of $k = \frac{ln3}{4}$. **2**

$P(4) = 300$

$300 = 100 \times e^{4k}$ — 1 mark: Correct working

$3 = e^{4k}$

$\therefore k = \frac{\ln 3}{4}$ — 1 mark: ~~show~~ Correct result

* This question was answered well

(b) Determine the population of bacteria after 12 hours. **2**

$P(12) = 100e^{12 \times \frac{\ln 3}{4}}$ — 1 mark: Correct working

$= 100 \times e^{\ln 3^3}$ — 1 mark: Correct answer

$= 2700$

* Most students answered this correctly

(c) Find the time it takes for the population to reach 100,000 bacteria. Leave your answer to the nearest minute. **2**

$100\,000 = 100e^{\frac{t \ln 3}{4}}$ — 1 mark: Correct working

$1000 = e^{\ln 3 \frac{t}{4}}$ — 1 mark: Correct answer

$1000 = 3^{\frac{t}{4}}$

$\log_3 1000 = \frac{t}{4}$

$t = 4 \times \frac{3 \log 10}{\log 3}$ $\quad t \approx 25.15$

$= 4 \times \frac{3}{\log 3}$ $\quad = 25 + 0.15 \times 60$

$= 25$ hrs and 9 mins

* Some students converted the decimal to time incorrectly

Do NOT write in this area.

Question 20 (3 marks)

Given the function $f(x) = 2x^3 - \frac{5}{2}x^2 - 4x + 2$. Determine the interval(s) where $f(x)$ is decreasing. **3**

$f'(x) = 6x^2 - 5x - 4$

$f'(x) = 0$

$x = \frac{5 \pm \sqrt{25 + 4 \times 6 \times 4}}{12}$

$= \frac{5 \pm \sqrt{121}}{12}$

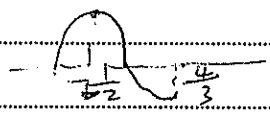
$= \frac{5 \pm 11}{12}$

$\therefore x = \frac{16}{12} = \frac{4}{3}$ or $x = -\frac{1}{2}$

$f''(x) = 12x - 5$

$f''(-\frac{1}{2}) = -6 - 5 = -11 < 0$

$f''(\frac{4}{3}) = 16 - 5 = 11 > 0$



$\therefore x \in (-\frac{1}{2}, \frac{4}{3})$ where $f(x)$ is decreasing

or $-\frac{1}{2} < x < \frac{4}{3}$

* Most student differentiated correctly and found the stationary points 1 mark:

Finding correct stationary points with working 1 mark:

Correct determining the nature of stationary pts.

1 mark: Correct interval.

* Some students did not prove the ~~interval~~ function is decreasing between the interval.

Question 21 (3 marks)

Given the following two functions $f(x) = x^2 - 3$ and $g(x) = \sqrt{2 - x}$. **3**

Find the domain and range of $f(g(x))$.

$f(\sqrt{2-x}) = 2 - x - 3 = -x - 1$

domain: $2 - x \geq 0$, $\therefore x \leq 2$

range: $[-3, \infty)$

* Most students found $f(g(x))$ but did not find the correct domain and range.

1 mark: Correct composite fn.

1 mark: Correct domain.

1 mark: Correct range.

$\sin^2\theta + \cos^2\theta = 1$

$1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$\tan^2\theta + 1 = \sec^2\theta$

Question 22 (5 marks)

Criteria	Mark
Provide correct solutions	2
Use trigonometric identities to simplify LHS to $1+\frac{1}{\tan^2 2\theta}$ or $\frac{1}{1-\cos^2 2\theta}$	1

(a) Prove

$$\frac{\sec^2(2\theta)}{\sec^2(2\theta)-1} = \operatorname{cosec}^2(2\theta).$$

$LHS = \frac{\tan^2\theta + 1}{\tan^2\theta}$

$= 1 + \frac{1}{\tan^2\theta}$

$= 1 + \cot^2\theta$

$= \operatorname{cosec}^2\theta$

$= RHS$

$LHS = \frac{\frac{1}{\cos^2(2\theta)}}{\frac{1}{\cos^2(2\theta)} - 1}$

$= \frac{\frac{1}{\cos^2(2\theta)}}{\frac{1-\cos^2(2\theta)}{\cos^2(2\theta)}}$

$= \frac{1}{1-\cos^2(2\theta)}$

$= \frac{1}{\sin^2(2\theta)}$

$= \operatorname{cosec}^2(2\theta)$

$= RHS$

(b) Hence, solve $\frac{\sec^2(2\theta)}{\sec^2(2\theta)-1} = 2$ where $0 \le \theta \le \pi$. 3

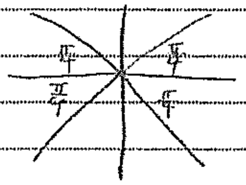
$0 \le 2\theta \le 2\pi$

$\operatorname{cosec}^2(2\theta) = 2$

$\operatorname{cosec}(2\theta) = \pm\sqrt{2}$

$\frac{1}{\sin(2\theta)} = \pm\sqrt{2}$

$\sin(2\theta) = \pm\frac{1}{\sqrt{2}}$



$2\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$\theta = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}$

Do NOT write in this area.

Part (a) was done well by all the students. The most common error for Part (b) was that many students either forgot to take the $\pm$ into consideration when squar rooting $\sin^2(2\theta)$ or forgot to change the domain to $0 \le 2\theta \le 2\pi$

18

Criteria	Mark
Provide correct solutions	3
Change the domain to $0 \le 2\theta \le 2\pi$	2
Use part (a) to simplify the equation into $\sin(2\theta) = \pm\frac{1}{\sqrt{2}}$	1

Question 23 (4 marks)

A ship sets sail from point P on a bearing of 060° for 150 km to point Q. It then changes course and sails on a bearing of 160° for 200 km to point R.

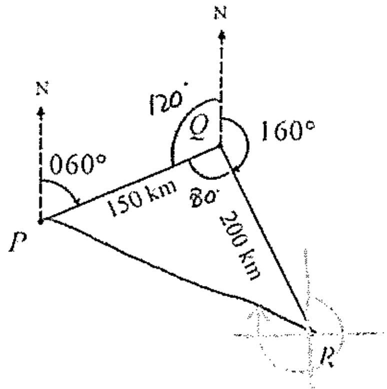


Diagram not to scale

(a) Determine the distance from point P to point R. Correct your answer to 2 decimal places. **2**

$PR = \sqrt{150^2 + 200^2 - 2(150)(200)\cos 80^\circ}$

$= 228.21$

Criteria	Mark
Provide correct solutions	2
Show that $\angle PQR = 80°$ or apply the cosine rule	1

(b) Determine the bearing the ship must take to return directly to point P from point R. Correct your answer to the nearest degree. **2**

$\cos \angle QRP = \dfrac{200^2 + PR^2 - 150^2}{2(QR)(PR)}$

$\angle QRP = 40.33800\ldots$

Bearing $= 360 - \angle QRP - 20$

$= 299.6619\ldots$

$= 300^\circ$

Criteria	Mark
Provide correct solutions	2
Show that $\angle QRP = 40°$	1

- Part (a) was most done well by most, however there were a handful of students found the incorrect value of $\angle PQR$
- Many found the bearing of R from P instead of P from R

Question 24 (5 marks)

A financial analyst is studying the relationship between the number of years of experience, n (in years) and the salary, s (in thousands of dollars) of employees in a certain industry. The analyst collects data from a sample of employees and records their years of experience and corresponding salaries as follows:

Years of Experience (n)	Salary (s) (in \$1000s)
1	65
3	70
5	80
7	100
9	130

Criteria	Mark
Provide correct solutions	2
Find the correct value of a or b	1

(a) Calculate the least squares regression line for this data. 2

$a = 49$

$\therefore y = 8x + 49$

$b = 8$

Do NOT write i

Criteria	Mark
Provide correct solutions	2
Correct interpret the slope or the intercept	1

(b) Interpret the slope and intercept of the regression line in the context of this problem.

- On average, one year increase in expevicnee the salary increases by \$8000.

- The starting Salary is \$44000

(c) Why is this line NOT useful for predicting the salary for a person who has 12 years of work experience?

Can't assume that salary increases at the same rate after year 9.

Criteria	Mark
Provide correct solutions	1

- **Part (a) was done well by everyone**
- **Part (b) was done well by most, but a handful of student interpreted correlation coefficient instead**
- **Part (c) was done well**

Do NOT write in this area.

Question 25 (3 marks)

The scores of a Year 11 economics examination are shown in the back-to-back stem and leaf plot below for classes 11A and 11B. 3

11 A		11 B
8 7 5	5	9
9 8 8 8 4 2	6	4 4
7 5 2	7	0 3 5 7
3 1	8	2 5 6 7 8
4	9	1 2
15		14

Positively Skewed.

negative Skewed.

Mrs Cartwright claims that class 11B did better in the examination than class 11A.

Do you agree with Mrs Cartwright? Justify your answer by referring to the median and skewness of the two sets of scores.

Median 11A : 68

Median 11B : $\frac{77+82}{2} = 79.5$

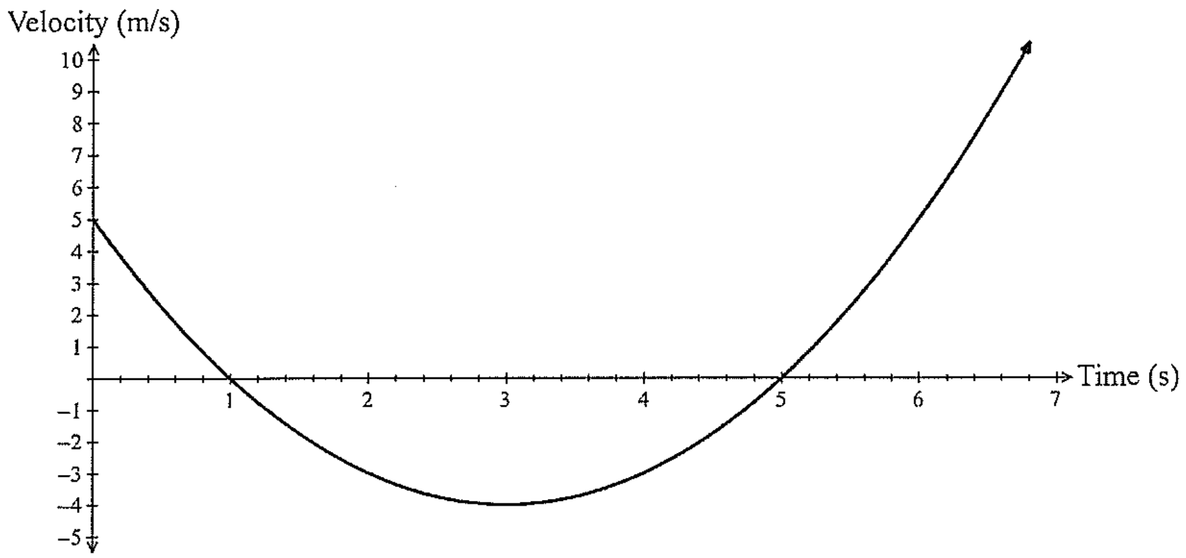
Mrs Cartwright is correct because the median of 11B is higher than 11A. Also 11B's results are negatively skewed, meaning that 11B's results are higher.

Criteria	Mark
Provide correct solutions	3
Correct calculate the median of both class and correctly identify the skewness of both class.	2
Correct calculate the median of both class or correctly identify the skewness of both class.	1

Many student found the incorrect median for class 11A or 11B and they also incorrectly identified the skewness of both class.

Question 26 (5 marks)

The velocity-time graph shows how a particle travels during a period of 7 seconds. Initially, the particle is at the origin and travelling at 5 m/s to the right. The graph has two horizontal intercepts at $t = 1$ and $t = 5$. It also has a turning point at $t = 3$.



(a) Explain the meaning of the horizontal intercept at $t = 1$. **1**

The particle is at rest.

Feedback: Some students mentioned the particle changed direction without stating that it was stationary

(b) When the acceleration is not zero, what is the direction of the acceleration? **2**

~~at t = 3~~

From t=0 to t=3s, acceleration is to the left, ~~from t=~~
From t=3s to t=7s, acceleration is to the right.

1 mark: Correct describe acceleration for one interval

1 mark: Correct describe acceleration for both intervals

Feedback: Some students should learn correct terminology. Most did not separate the two cases.

Question 26 continues page 23

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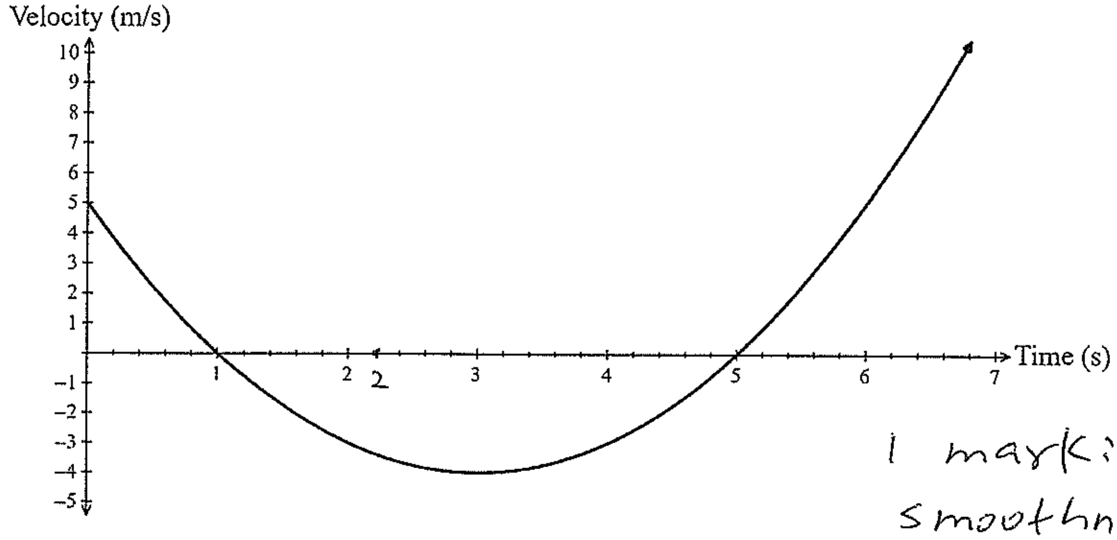
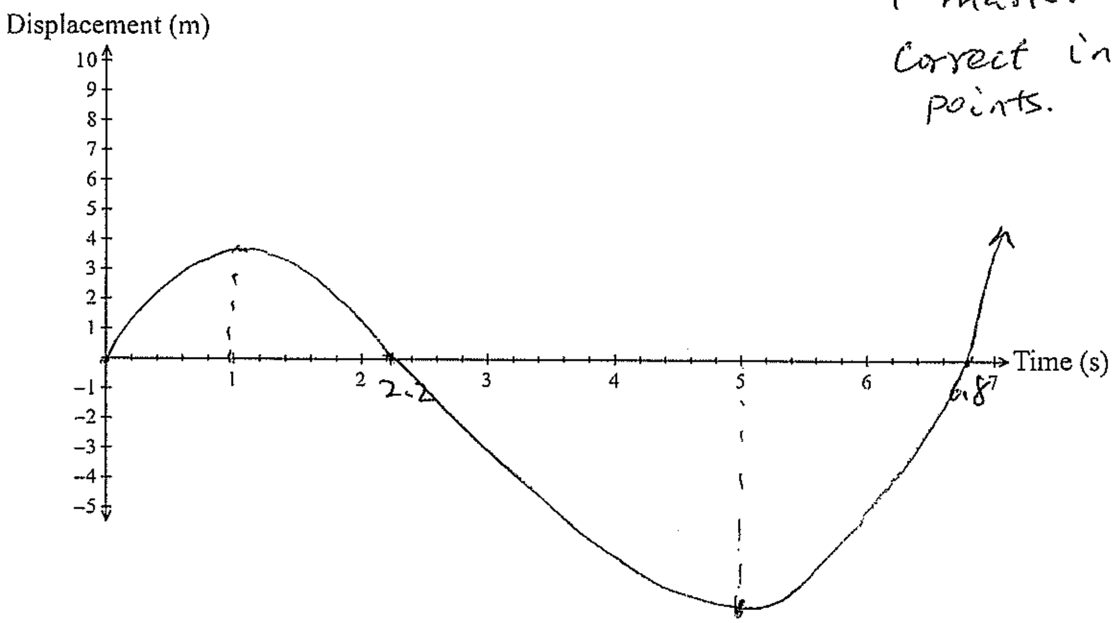
Question 26 (continued)

(c) Hence sketch the displacement-time graph below. The displacement is 0 when $t = 2.2\ sec$ and $t = 6.8\ sec$. **2**



1 mark: smoothness of the curve.

1 mark: Correct intercept points.



Feedback: It is important to clearly indicate the stationary points at $t=1$ and $t=5$. Some students did not start from the origin.

Question 27 (3 marks)

Solve the following equation: **3**

$$x - 2x^2 + x^3 - 2x^4 + x^5 - 2x^6 + \cdots = -\frac{2}{5}$$

Where $-1 \le x \le 1$

$a = x - 2x^2, \quad T_2 = x^3 - 2x^4$

$T_3 = x^5 - 2x^6$

$r = \frac{x^3 - 2x^4}{x - 2x^2} = \frac{x^3(1-2x)}{x(1-2x)} = x^2$ — 1 mark: Correct ratio with working

since $-1 \le x \le 1$

$r < 1$

$\therefore$ The series is limited GP. — 1 mark: Correct limit Sum of GP

$S = \frac{a}{1-r}$

$= \frac{x - 2x^2}{1 - x^2}$ — 1 mark: Correct x-values.

$\frac{x - 2x^2}{1 - x^2} = -\frac{2}{5}$

$5x - 10x^2 = -2 + 2x^2$

$12x^2 - 5x - 2 = 0$

$(3x - 2)(4x + 1) = 0$

$\therefore x = -\frac{2}{3}$ or $x = -\frac{1}{4}$

Feedback: The question has been done poorly. A number of students did not recognise this is a geometric series.

End of Booklet 1 Section II

Do NOT write in this area.

Question 28 (2 marks)

Given that $\frac{dy}{dx} = \cos\left(x - \frac{\pi}{4}\right)$ and $y = 2$ when $x = \frac{3\pi}{4}$, find y in terms of x. **2**

$\int \cos\left(x - \frac{\pi}{4}\right) dx = \left[\sin\left(x - \frac{\pi}{4}\right)\right] + c$ 1 mark for correct integration

when $y = 2$, $x = 3\pi/4$: $2 = \sin\left(\frac{3\pi}{4} - \frac{\pi}{4}\right) + c$

$2 = \sin\left(\frac{\pi}{2}\right) + c$

$c = 1$

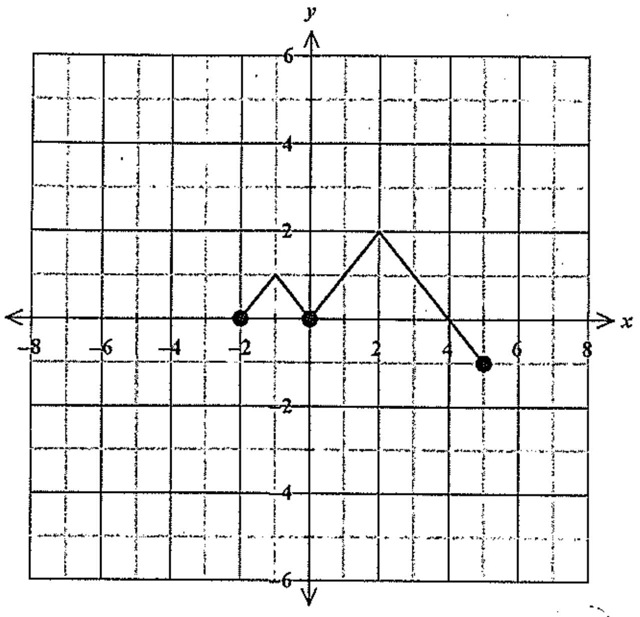
$\therefore y = \sin\left(x - \pi/4\right) + 1$ 1 mark for correct solution

This question has done well.
A small number of students could not integrate cosine or ~~did not rem~~ figure out the exact value.

Do NOT write in this area.

Question 29 (2 marks)

Given the following function $f(x)$.

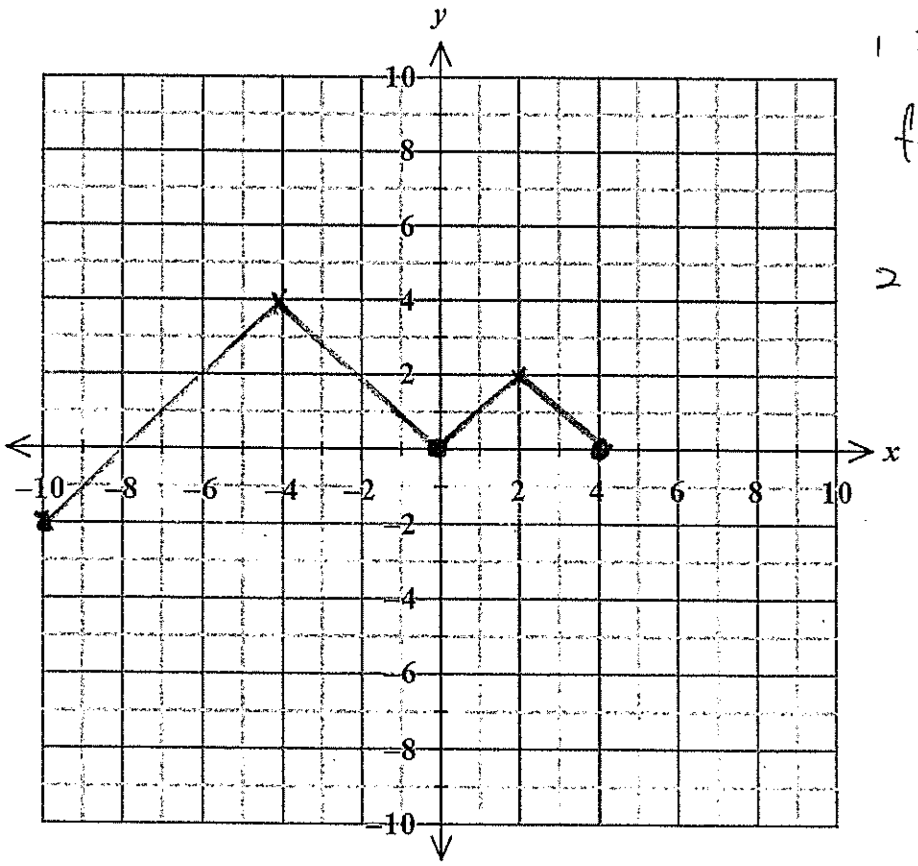


1 mark for

Sketch $2f\left(-\frac{1}{2}x\right)$ on the axis below.

1 mark for $2f(\frac{1}{2}x)$

2 marks for correct answer.



Feed back: A lot of students were confused when doing $f(\frac{1}{2}x)$. They compressed the graph instead of extending it horizontally.

Question 30 (6 marks)

The quantity Q in mL of a certain chemical in the body varies during the day and is given by the formula $Q(t) = 4 + 3\cos\left(\frac{\pi t}{4}\right)$

where t measures hours from midnight.

(a) Find the period in hours of the function Q. **1**

Most students did well in part (a)

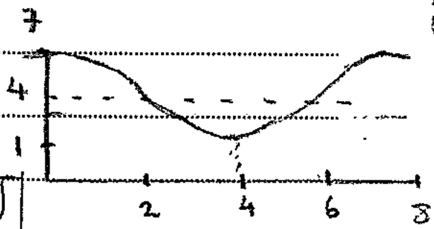
$T = \frac{2\pi}{\pi/4}$

$= 8$ hours   1 Mark for correct solution

(b) At what time or times of the day is the quantity a minimum? **2**

$Q'(t) = 0$

$\therefore t = 4$ hours



① mark for graph

Most students did not provide graph for part (b)

Times of the day = 4am, 12pm and 8pm ① Mark for all three solution

(c) What is the minimum quantity of the chemical the body will contain? **1**

$Q(4) = 4 + 3\cos\pi$

$= 4 - 3$

$= 1$ mL

Most students did well!

(d) A hospital patient requires a pill to temporarily boost the amount of the chemical whenever the quantity in his body falls to 1.5 units. At what time will a nurse have to wake the patient to give him his first pill of the day? Give your answer to nearest minute. **2**

S | A
T | C

$1.5 = 4 + 3\cos\left(\frac{\pi t}{4}\right)$

$\frac{-2.5}{3} = \cos\left(\frac{\pi t}{4}\right)$ ① Mark for making $Q(t) = 1.5$

$\frac{\pi t}{4} = \pi - \cos^{-1}\left(5/6\right) = 2.5559...$

$t = 2.5559 \times \frac{4}{\pi}$

$= 3.254...$

$= 3$h 15 mins   ① Mark for correct solution

Some students did not convert 3.254 to 3h 15min.

Question 31 (4 marks)

Given that $y = x^2 \ln(x)$

(a) Show that **2**

$$\frac{dy}{dx} = 2x\ln(x) + x$$

$$\frac{dy}{dx} = 2x\ln(x) + x^2 \times \frac{1}{x}$$

$$= 2x\ln(x) + x$$

1 mark for correct working

well done!

1 mark for correct answer

(b) Hence, show that **2**

$$\int x\ln(x)\,dx = \frac{x^2\ln(x)}{2} - \frac{x^2}{4} + C$$

$$\Rightarrow \int 2x\ln(x) + x\,dx = x^2\ln(x)$$

$$\Rightarrow 2\int x\ln(x)\,dx + \int x\,dx = x^2\ln(x)$$

$$\Rightarrow 2\int x\ln(x)\,dx = x^2\ln x - \int x\,dx$$

$$\int x\ln(x)\,dx = \frac{x^2\ln(x)}{2} - \frac{1}{2}\int x\,dx$$

$$= \frac{x^2\ln(x)}{2} - \frac{1}{2}\cdot\frac{x^2}{2}$$

$$= \frac{x^2\ln(x)}{2} - \frac{x^2}{4} + C$$

Some students did not know how to show left hand side = right hand side. They just copied the answer from right hand side.

1 mark for correct working

1 mark for correct answer

Question 32 (4 marks)

The first two terms of an infinite geometric sequence are $T_1 = 20$ and $T_2 = 16\sin^2\theta$, where $0 < \theta < 2\pi$, $\theta \neq \pi$.

(a) Find the range of the ratio (r) in this geometric sequence. **2**

$\frac{T_2}{T_1} = \frac{16\sin^2\theta}{20} = r$

$r = \frac{4}{5}\sin^2\theta$ when $0 < \theta < 2\pi$, $\theta \neq \pi$

$0 < r \leq \frac{4}{5}$

OR $r \in [0, \frac{4}{5}]$.

1 mark for correct working
1 mark for correct range.

Many students did not provide the range of ratio r.

(b) By developing an expression for the sum of the infinite sequence, find the values of θ which give the greatest sum. **2**

$S = a\left(\frac{1}{1-r}\right)$

$= 20\left(\frac{1}{1-\frac{4}{5}\sin^2\theta}\right)$

$= \frac{20\times 5}{5-4\sin^2\theta}$

$= \frac{100}{5-4\sin^2\theta}$

$\frac{ds}{d\theta} = \frac{-100(8\sin\theta\cos\theta)}{(5-4\sin^2\theta)^2}$

when $\frac{ds}{d\theta} = 0$ $\theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ $(0 < \theta < 2\pi)$ $\theta \neq \pi$

θ	$\frac{\pi}{2} - 0.01$	$\frac{\pi}{2}$	$\frac{\pi}{2} + 0.01$
S	99.96	100	99.96

θ	$\frac{3\pi}{2} - 0.01$	$\frac{3\pi}{2}$	$\frac{3\pi}{2} + 0.01$
S	99.96	100	99.96

$\therefore$ max at $\theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$

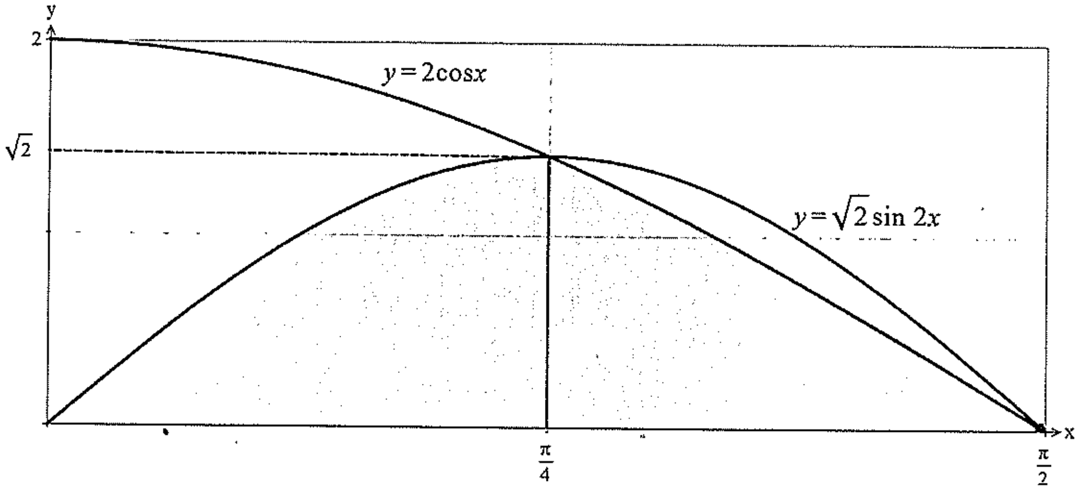
Many students did not find the values of θ which give the greatest sum

1 mark for correct working
1 mark for correct answer

Question 33 (3 marks)

The diagram below shows the graphs of the functions $y = 2\cos x$ and $y = \sqrt{2}\sin 2x$ between $x = 0$ and $x = \frac{\pi}{2}$. **3**

The two graphs intersect at $x = \frac{\pi}{4}$ and $x = \frac{\pi}{2}$.



Find the area of the shaded region. Leave your answer in exact form.

$$A = \int_0^{\frac{\pi}{4}} \sqrt{2}\sin 2x\,dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 2\cos x\,dx$$

$$= \left[-\frac{\sqrt{2}}{2}\cos 2x\right]_0^{\frac{\pi}{4}} + \left[2\sin x\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \left[-\frac{\sqrt{2}}{2}\times 0 - \left(-\frac{\sqrt{2}}{2}\right)\right] + \left[2\times 1 - 2\times\frac{1}{\sqrt{2}}\right]$$

$$= \frac{1}{\sqrt{2}} + 2 - \sqrt{2}$$

$$= \frac{\sqrt{2}}{2} + \frac{4}{2} - \frac{2\sqrt{2}}{2}$$

$$= \frac{4-\sqrt{2}}{2}$$

3 marks for provide correct solutions,
2 marks for provide correct working and wrong answer
1 mark for correct answer.

Some students used the lower bound as zero to the upper bound as $\frac{\pi}{2}$ instead of 2 sections from zero to $\frac{\pi}{4}$ and $\frac{\pi}{4}$ to $\frac{\pi}{2}$ for two given functions.

Question 34 (3 marks)

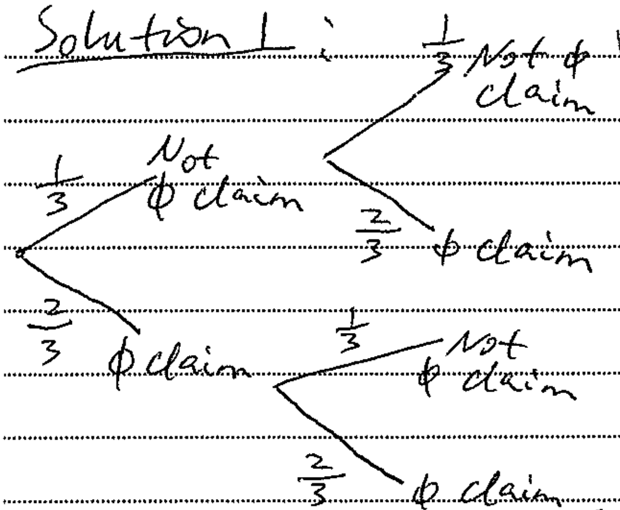
A flood insurance company determines that N, the number of claims received in a month, is a random variable with **3**

$$P(N = n) = \frac{2}{3^{n+1}}, \text{ for } n = 0, 1, 2, \ldots$$

The numbers of claims received in different months are independent.

In any consecutive two-month period, calculate the probability that more than one claim will be received, given that zero claims were received at least one of the two months.

Solution 1:



$P(N=0, N=0) = \frac{2}{3} \times \frac{2}{3} = \frac{8}{9}$

$P(\text{Not}(N=0, N=0)) = 1 - \frac{8}{9} = \frac{1}{9}$

$\therefore P(N > 1 / \text{at least } \phi \text{ claim in two months})$

(1st month, 2nd month)

$= \dfrac{\frac{1}{9} \times \frac{2}{3} + \frac{2}{3} \times \frac{1}{9}}{\frac{2}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{2}{3} + \frac{2}{3} \times \frac{2}{3}}$

$= \dfrac{\frac{4}{27}}{\frac{8}{9}}$

$= \frac{1}{6}$

Solution 2:

$P[N > 1 / (P(0,0) + P(0,n) + P(n,0))]$

$= 1 - [P(N=0) + P(N=1)]$

$= 1 - \dfrac{P(0)P(0) + P(0)P(1) + P(1)P(0)}{P(0,0) + P(0,n) + P(n,0)}$

$= 1 - \dfrac{\frac{2}{3} \times \frac{2}{3} + \frac{2}{3} \times \frac{2}{9} + \frac{2}{9} \times \frac{2}{3}}{\frac{2}{3} \times \frac{2}{3} + \frac{2}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{2}{3}}$

($P(N \neq 0)$)

$= 1 - \dfrac{\frac{4}{9} + \frac{8}{27}}{\frac{4}{9} + \frac{2}{9} \times 2}$

$= 1 - \frac{5}{6}$

$= \frac{1}{6}$

1 mark: Correct recognise sum of three combination events on the numerator

~~1 mark: or~~

1 mark: Correct working

1 mark: Correct answer.

* Done poorly.
One mark given for Correct substitution for $P(N=0) = \frac{2}{3}$ and $P(N=1) = \frac{2}{9}$

Question 35 (5 marks)

Melinda visits a bank and makes a single deposit of $\$Q$. The annual interest rate is 3.5%.

(a) Melinda wishes to withdraw \$8000 at the end of each year for a period of n years. Show that an expression for the minimum value of Q is **3**

$$\frac{8000}{1.035}+\frac{8000}{1.035^2}+\frac{8000}{1.035^3}+\cdots+\frac{8000}{1.035^n}$$

$A_1 = Q\times 1.035 - 8000$

$A_2 = A_1 \times 1.035 - 8000$

~~$= (Q\times 1.035 - 8000)1.035 - 8000$~~

$= Q\times 1.035^2 - 8000\times 1.035 - 8000$

$A_3 = A_2\times 1.035 - 8000$

$= Q\times 1.035^3 - 8000\times 1.035^2 - 8000\times 1.035 - 8000$

$= Q\times 1.035^3 - 8000(1.035^2 + 1.035 + 1)$

$\vdots$

$A_n = Q\times 1.035^n - 8000(1+1.035+\ldots+1.035^{n-1})$

~~$= Q\times 1.035^n - 8000\left(\frac{1.035^n-1}{0.035}\right)$~~

Minimum value of Q:

$Q\times 1.035^n = 8000(1+1.035+1.035^2+1.035^3+\ldots+1.035^{n-1})$

$Q = \frac{8000}{1.035^n}+\frac{8000}{1.035^{n-1}}+\frac{8000}{1.035^{n-2}}+\ldots+\frac{8000}{1.035}$

$= \frac{8000}{1.035}+\frac{8000}{1.035^2}+\ldots+\frac{8000}{1.035^n}$

* A few students ~~did not generate~~ try to use the results to express $A_1, A_2 \cdots A_n$

Criteria	Mark
Provide correct solutions	3
Find the correct expression for A_n	2
Write the correct expression for A_1, A_2 or A_3	1

Do NOT write in this area.

DO NOT write in this area.

Question 35 (continued)

(b) Hence, or otherwise, find the minimum value Q that would permit Melinda to withdraw annual amounts of \$8000 indefinitely. Give your answer to the nearest dollar. **2**

$$\frac{8000}{1.035} + \frac{8000}{1.035^2} + \ldots + \frac{8000}{1.035^n}$$

$$a = \frac{8000}{1.035}, \quad r = \frac{1}{1.035}$$

$$\text{Minimum value} = \frac{\frac{8000}{1.035}}{1 - \frac{1}{1.035}}$$

$$= \$228571.43$$

Alternative solution

minimum value × 3.5% = 8000

$$\therefore \text{Minimum value} = \frac{8000}{3.5\%}$$

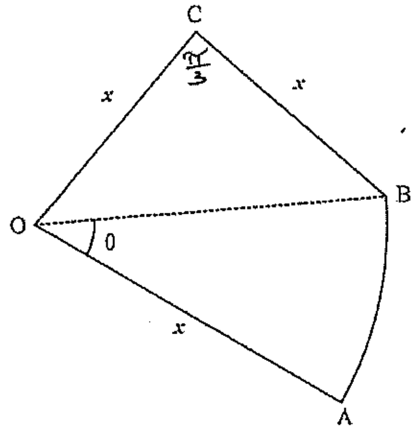
$$= \$228571.43$$

*Done well in general.

Criteria	Mark
Provide correct solutions	2
Using the correct limiting sum expression to find the minimum value	1

Question 36 (5 marks)

The diagram below shows a large bio-diversity precinct the council is planning to build inside a large park. OAB is a sector with centre O, and radius x kilometres. Arc AB subtends an angle of θ radians at O. The equilateral triangle BCO adjoins the sector. 5



The perimeter of the precinct as shown in the diagram is given to be $(12 - 2\sqrt{3})$ kilometres.

Calculate the maximum area of the precinct ($OABC$). Leave your answer in exact form.

Arc AB $= 12 - 2\sqrt{3} - 3x$

$\therefore \quad 12 - 2\sqrt{3} - 3x = x\theta$

$\theta = \frac{12}{x} - \frac{2\sqrt{3}}{x} - 3$

Area $\triangle OBC = \frac{1}{2} \times x^2 \times \sin\left(\frac{\pi}{3}\right)$

$= \frac{1}{2} \times x^2 \times \frac{\sqrt{3}}{2}$

Area of $\triangle OBC = \frac{x^2\sqrt{3}}{4}$

Area of Sector : $\frac{1}{2} \times x^2 \times \theta = \frac{x^2}{2}\theta$

Extra writing space is provided on page 39

* Quite a few students ~~did~~ left $\sin(60^\circ)$ as part of Area of $\triangle OBC$.

* A few students forgot to include Area of $\triangle OBC$ as part of total area.

Do NOT write in this area.

Question 36 (extra writing space)

Area of triangle + Area of Sector

$$\frac{x^2\sqrt{3}}{4} + \frac{x^2}{2}\left(\frac{12}{x} - \frac{2\sqrt{3}}{x} - 3\right)$$

$$= \frac{x^2\sqrt{3}}{4} + 6x - x\sqrt{3} - \frac{3x^2}{2}$$

$$A = \frac{(\sqrt{3}-6)x^2}{4} + (6-\sqrt{3})x$$

$$\frac{dA}{dx} = \frac{(\sqrt{3}-6)x}{2} + (6-\sqrt{3}) = 0$$

$$\frac{\sqrt{3}-6}{2}x = -(6-\sqrt{3})$$

$$x = \frac{-2(6-\sqrt{3})}{\sqrt{3}-6}$$

$$x = \frac{2(\sqrt{3}-6)}{\sqrt{3}-6}$$

$$x = 2$$

Since $\frac{d^2A}{dx^2} = \frac{\sqrt{3}-6}{2} < 0$, then $x=2$ is maximum.

$$\therefore \text{Total area} = \frac{4(\sqrt{3}-6)}{4} + 2(6-\sqrt{3})$$

$$= 6-\sqrt{3} \text{ km}^2$$

Criteria	Mark
Provide correct solutions	5
Find the expression of $\frac{dA}{dx}$, or equivalent merit	4
Find the correction expression of the total area of the precinct	3
Find the correction expression for the area of the triangle or the area of the sector in terms of x or θ	2
Find the correct expression of θ in terms of x, or	1